DETERMINATION OF THE RHEOLOGICAL CHARACTERISTICS OF EPOXY RESIN IN DYNAMIC TESTS

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A survey of investigations of the rheological characteristics of materials at high deformation rates can be found in [1-4], and we shall, therefore, limit our considerations to the basic results. It was found in [5] that there is a difference between the yield points in static and dynamic tests, which subsequently led to the concept of the dynamic tensile—compressive diagram for specimens in the plastic region, which was first defined in [6]. Experiments performed with greater accuracy were discussed in [7], where the residual strain produced by the collision of rods was determined, while the dynamic diagram was determined by means of the method described in [8].

It was discovered in [9, 10], apparently for the first time, that waves propagate at elastic velocities in copper not only in the elastic region, but also beyond the yield point. These results were subsequently confirmed by more refined experiments [11, 12] on different materials, which indicated the necessity of accounting for the effect of the plastic strain rate in investigating wave processes. From among the many papers devoted to the effect of the strain rate effect, the investigations described in [13, 14], where aluminum and steel specimens were tested in a complex stressed state, are of interest. It was shown there that, in loading, at a high rate, the second invariant of the stress tensor J_2 depends not only on the second invariant of the strain tensor I_2, but also on $\mathring{I}_2$.

The general relationships for rate sensitive media have been proposed in [15, 16], where the relationships given in [9] are generalized to include a complex stress state. Another variant of generalization of Malvern's relationships is proposed in [17]. The theories set forth in [15-17] comprise certain functions which must be determined experimentally. Certain approximations have been found in [15] for these functions on the basis of experimental data [18], where the yield point of soft steel as a function of the strain rate is determined in the range $0 \leqslant \dot{\varepsilon} \leqslant 200\ \text{sec}^{-1}$. The trends of wave propagation in media sensitive to the plastic strain rate are investigated in [19]. We propose here a method of measuring the rheological characteristics of materials based on the solution obtained in [19]. This method has been developed in the laboratories of the Kuibyshev State University.

The total strain is composed of the elastic and the plastic components:

$$e_{ij} = e_{ij}^{e} + e_{ij}^{p}. \tag{1}$$

The elastic components e_{ij}^{e} are related to the stress by Hooke's law,

$$\sigma_{ij} = \lambda e_{kk}^{e}\delta_{ij} + 2\mu e_{ij}^{e}, \quad e_{ij}^{e} = \frac{1}{2\mu} s_{ij} + \frac{1-2\nu}{E}\sigma_{kk}\delta_{ij}, \tag{2}$$

where λ and μ are Lamé constants; $s_{ij} = \sigma_{ij} - (1/3)\sigma_{kk}\delta_{ij}$, the stress deviator.

For media sensitive to the plastic strain rate, we assume [20] that the loading surface depends on the plastic strain rate:

$$f\left(\sigma_{ij}, e_{ij}^{p}, \dot{e}_{ij}^{p}, \chi_i\right) = 0, \tag{3}$$

where χ_i are certain parameters of the plastic deformation history.

The plastic strain rate is determined on the basis of the associated law of flow:

$$\dot{e}_{ij}^{p} = \Psi \frac{\partial f}{\partial \sigma_{ij}}, \tag{4}$$

where Ψ is a positive indeterminate factor, the equation for which is obtained by substituting (4) in (3):

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$$f\left(\sigma_{ij}, e_{ij}^p, \Psi \frac{\partial f}{\partial \sigma_{ij}}, \chi_i\right) = 0. \tag{5}$$

If $\partial f/\partial \sigma_{ij}$ is independent of the plastic strain rate, we find from (5) that $\Psi = \Psi(\sigma_{ij}, e_{ij}^p, \chi_{ip})$, i.e., Ψ is independent of $\dot{e}_{ij}^p$. In this case, the results given in [15, 16] follow from the general relationships (1)-(4) if we put

$$\Psi = \gamma \langle \Phi(F) \rangle, \quad F = \frac{f(\sigma_{ij}, e_{ij}^p)}{\varkappa} - 1, \quad \varkappa = \varkappa\left(\int_0^t \sigma_{ij} de_{ij}^p\right). \tag{6}$$

The relationships proposed in [17] are obtained if the following is put in (1)-(5):

$$\Psi = \gamma \langle \Phi(F) \rangle \frac{1}{\sigma_i}, \quad F = \frac{\sigma_i}{f(e_i)} - 1, \quad f(\sigma_{ij}, e_{ij}^p, \dot{e}_{ij}^p) = \sigma_i - f(e_i) - \Phi^{-1}\left(\frac{\dot{e}_i^p}{\gamma}\right), \tag{7}$$

where γ is the viscosity coefficient; σ_i and e_i, intensities of the stress and strain deviators, $\dot{e}_i{}^p$, intensity of the plastic strain rate; Φ, $\langle \Phi(F) \rangle = \Phi(F)$ for $F > 0$, and $\langle \Phi(F) \rangle = 0$ for $F \leqslant 0$.

It is necessary to distinguish between two problems in experimental investigations of the relationship between stress and strain. The first problem involves the study of the material's behavior at an arbitrarily low strain rate, $\dot{e}_{ij} = 0$, i.e., the study of changes in the loading surface. In [15, 16], this problem in determining $f(\sigma_{ij}, e_{ij}^p)$, while, in [17], it involves determination of the dependence $\sigma_i = f(e_i)$. The theory in [17] is the isotropic hardening theory. The second problem consists in determining the effect of the strain rate, i.e., determining $\Phi(F)$ in the above theories. While the first problem is a familiar one and is solved by purely experimental means within the framework of plasticity theory, the solution of the second problem for high strain rates involves additional difficulties, connected primarily with the separation of the effect of inertial forces that develop in this case from the effect of the strain rate. For this separation, it is necessary to know the properties of solutions of the dynamic equations for elastoplastic media sensitive to the strain rate for sufficiently arbitrary rheological relationships, i.e., for arbitrary $f(\sigma_{ij}, e_{ij}^p, \dot{e}_{ij}^p, \chi_i)$ or arbitrary $\Phi(F)$ in [15-17]. Some of these properties were investigated in [19], where the function $f(\sigma_{ij}, e_{ij}^p, \dot{e}_{ij}^p, \chi_i)$ was used in the following form:

$$(s_{ij} - c e_{ij}^p - \eta \dot{e}_{ij}^p)(s_{ij} - c e_{ij}^p - \eta \dot{e}_{ij}^p) - \frac{2}{3} k^2 = 0. \tag{8}$$

It is convenient to assume here that c and k are functions of e_{ij}^p and χ_i, which are determined in tests at low loading rates, and that η is a function of the plastic strain rate $\dot{e}_i^p = \sqrt{\dot{e}_{ij}^p \dot{e}_{ij}^p}$, while k is the tensile yield point. It has been shown in [19] that only two types of waves, which propagate at the elastic velocities $\rho c_1^2 = \lambda + 2\mu$, and $\rho c_2^2 = \mu$, can occur in rate sensitive materials. This fact is in agreement with experimental data [9-12]. The following relationships hold for waves propagating at the velocity c_1:

$$[v_i] = \omega \nu_i, \; c_1[\sigma_{ij}] = -\omega(\lambda \delta_{ij} + 2\mu \nu_i \nu_j), \tag{9}$$

where ν_i is the normal to the wave front and ω is the intensity of the wave, whose variation in motion along the normal is described by the differential equation

$$\frac{\delta \omega}{\delta t} = c_1 \Omega \omega + \frac{\mu}{\rho c_1}\left[\dot{e}_{ij}^p\right] \nu_i \nu_j, \tag{10}$$

where $[v_i]$, $[\sigma_{ij}]$, and $[\dot{e}_{ij}^p]$ are the discontinuities in the displacement rate, stress, and the plastic strain rate behind the wave front, and Ω is the mean curvature of the wave surface.

The following relationships hold for waves propagating at the velocity c_2:

$$[\sigma_{ij}] = -\rho c_2([v_i]\nu_j + [v_j]\nu_j), \; [v_k]\nu_k = 0. \tag{11}$$

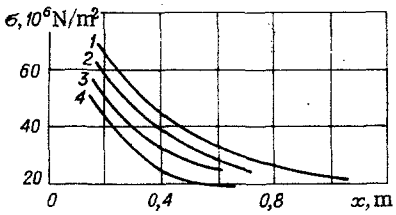

Fig. 1

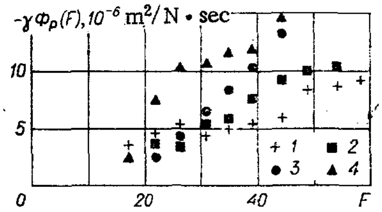

Fig. 2

The value of $[v_i]$ varies in motion along the normal to the wave surface in accordance with the equations

$$\frac{\delta [v_i]}{\delta (t)} = c_2\Omega [v_i] + c_2([\dot{e}_{ij}^p] \nu_j - [\dot{e}_{kl}^p] \nu_k\nu_l\nu_i). \tag{12}$$

Wave (9) is the easiest one to produce experimentally, and, therefore, we shall analyze only the case where the wave under consideration propagates in a stationary medium at the velocity c_1. The plastic strain behind the wave front is equal to zero, and, for the Perzyna theory [15, 16], we obtain the plastic strain rate in the following form:

$$\dot{\varepsilon}_{ij}^p = \gamma\Phi_P(F)\, s_{ij}, \tag{13}$$

where $F = -1 + \sqrt{3s_{ij}s_{ij}/2k^2}$, i.e., in the initial state, the yield surface constitutes the Mises circle. For Kaliski's theory, we obtain

$$\dot{\varepsilon}_{ij}^p = \gamma\Phi_K(F)\, s_{ij}/\sqrt{s_{kl}s_{kl}}. \tag{14}$$

It is evident from (13) and (14) that the functions $\Phi_P(F)$ and $\Phi_K(F)$ figuring in both theories are related by the expressions

$$(F + 1)\sqrt{\frac{2}{3}}\, k\Phi_P(F) = \Phi_K(F). \tag{15}$$

Therefore, we shall subsequently limit our considerations to relationships (13).

Assuming that the loading surface is given by (8), we obtain from (3) and (4) the following equation for the plastic strain rate:

$$s_{ij} = \frac{\sqrt{\frac{2}{3}}\, k\dot{e}_{ij}^p}{\sqrt{\dot{e}_{kl}^p\dot{e}_{kl}^p}} + \eta\dot{e}_{ij}^p. \tag{16}$$

Relationships (16) define the stress deviator in terms of the plastic strain rate, while relationships (13), conversely, define the plastic strain rate in terms of the stress. Since $\sigma_{ij} = 0$ ahead of the wave front, it follows from (9) that the following expressions hold behind the wave front:

$$\sigma_{ij} = \frac{\omega}{c_1}(\lambda\delta_{ij} + 2\mu\nu_i\nu_j), \quad s_{ij} = \frac{2\mu\omega}{c_1}\left(\nu_i\nu_j - \frac{1}{3}\delta_{ij}\right), \quad \sigma_i = \frac{2\mu\bar{\omega}}{c_1}\sqrt{\frac{2}{3}}. \tag{17}$$

The discontinuities in the plastic strain rate for the model described in [15, 16] are determined on the basis of relationships (13), whence

$$[\dot{e}_{ij}^p] = -\gamma\Phi_P(F)\frac{2\mu\omega}{c_1}\left(\nu_i\nu_j - \frac{1}{3}\delta_{ij}\right), \quad F = \frac{2\mu\omega}{kc_1} - 1. \tag{18}$$

For the loading surface (8), the plastic strain rate behind the wave front is determined from relationships (16), whence

$$\frac{2\mu\omega}{c_1}\left(\nu_k\nu_l - \frac{1}{3}\delta_{kl}\right) = \frac{\sqrt{\frac{2}{3}}\, k\dot{e}_{kl}^p}{\dot{e}_i^p} + \eta(\dot{e}_i^p)\dot{e}_{kl}^p. \tag{19}$$

For determining $\dot{e}_i^p$, we obtain from (19) the equation

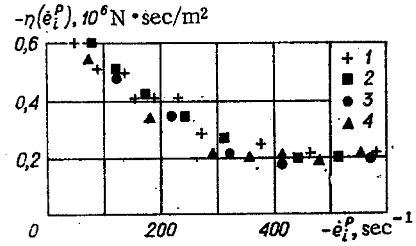

Fig. 3

$$\dot{e}_i^p\left(\sqrt{\frac{2}{3}}\frac{k}{\dot{e}_i^p}+\eta\left(\dot{e}_i^p\right)\right)=\sqrt{\frac{2}{3}}\frac{2\mu\omega}{c_1} \tag{20}$$

and, for determining the plastic strain, the expression

$$[\dot{e}_{kl}^p]=-\sqrt{\frac{3}{2}}\dot{e}_i^p\left(\nu_k\nu_l-\frac{1}{3}\delta_{kl}\right), \tag{21}$$

where $\dot{e}_i^p$ is expressed in terms of ω according to Eq. (20). If we produce experimentally a two-dimensional load wave in the material and measure $\omega(x)$, the wave intensity at the time the front passes through the point with the coordinate x, then, by using relationships (10), we obtain

$$\frac{d\omega}{dx}=\frac{\mu}{\rho c_1^2}[\dot{e}_{ij}^p]\nu_i\nu_j. \tag{22}$$

In processing experimental data according to the theory given in [15, 16], it is convenient to present the values of $d\omega/dx$ as functions of $F = 2\mu\omega/kc_1 - 1$. We then obtain from relationships (22) and (18)

$$\gamma\Phi_P(F)=-\frac{3\rho c_1^2}{2\mu k(F+1)}\frac{d\omega}{dx}. \tag{23}$$

In processing experimental data according to the theory involving the loading surface (8), it is convenient to plot ω as a function of $-\frac{\rho c_1^2}{\mu}\sqrt{\frac{3}{2}}\frac{d\omega}{dx}$, since we have the following from relationships (22) and (21):

$$\dot{e}_i^p=-\frac{\rho c_1^2}{\mu}\sqrt{\frac{3}{2}}\frac{d\omega}{dx}.$$

We then determine experimentally the value of ω as a function of $\dot{e}_i^p$ and obtain from (20)

$$\eta(\dot{e}_i^p)=\sqrt{\frac{2}{3}}\left(\frac{2\mu\omega(\dot{e}_i^p)-c_1k}{c_1\dot{e}_i^p}\right). \tag{24}$$

The experiments were performed on specimens of ED-6 epoxy resin, hardened by means of a maleic hardener. The specimens were prepared by pouring the mixture of resin and hardener into a glass tube with the pressure gauges fastened along the axis. The pressure gauges consisted of barium titanate tablets with a diameter of 3 mm and a thickness of 0.8 mm, which were first calibrated according to the method described in [21]. The specimens had a diameter of 40 mm and a length of 1200 mm. The load was applied to specimens by the shock wave produced as a result of electric detonation of an aluminum foil pasted on the specimen's endface. The foil was blasted by means of discharge from a battery consisting of six IK-100-0.25 capacitors through a mechanical impact spark gap with a peaker. The duration and the initial pressure in the pulse were varied by changing the foil thickness and commutating the capacitors in the battery. The signals from the gauges were recorded on an S1-33 oscilloscope. The accuracy of pressure measurements along the through channel was not worse than 10%. The position of the gauges along the specimen was measured with an accuracy of ±0.2 mm.

Figure 1 shows the variation of pulse pressure along the specimen in four cases: 1) discharge energy, 7.5 kJ; 2) 6.2 kJ; 3) 5.0 kJ; 4) 3.7 kJ. The pulse duration is the same: $2.5 \cdot 10^{-5}$ sec. The initial data for plotting relationships (23) and (24) are the following: $c_1 = 2060$ m/sec, $\rho = 1220$ kg/m^3, $k = 2.7 \cdot 10^6$ N/m^2, $c_2 = 1140$ m/sec, and $\nu = 0.39$. Figure 2 shows relationship (23), while Fig. 3 illustrates relationship (24). The markings of the calculation points correspond to four different cases of loading: 1-4) Discharge energies of 7.5, 6.2, 5, and 3.7 kJ. The considerably lesser scattering of values in Fig. 3 means that the results obtained by experimental data processing based on theory involving the load surface (8) are less sensitive to errors in determining the pulse pressure.

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